

FCNC B_s and Λ_b transitions:

Standard Model versus a single Universal Extra Dimension scenario

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We study the FCNC $B_s \rightarrow \phi\gamma, \phi\nu\bar{\nu}$ and $\Lambda_b \rightarrow \Lambda\gamma, \Lambda\nu\bar{\nu}$ transitions in the Standard Model and in a scenario with a single Universal Extra Dimension. In particular, we focus on the present knowledge of the hadronic uncertainties and on possible improvements. We discuss how the measurements of these modes can be used to constrain the new parameter involved in the extra dimensional scenario, the radius R of the extra dimension, completing the information available from B-factories. The rates of these $b \rightarrow s$ induced decays are within the reach of new experiments, such as LHCb.

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I. INTRODUCTION

The heavy flavour Physics programmes at the hadron facilities, the Tevatron at Fermilab and the Large Hadron Collider (LHC) at CERN, include the analysis of heavy particles, in particular B_s and Λ_b , which cannot be produced at the e^+e^- factories operating at the peak of $\Upsilon(4S)$ [1]. These programmes involve as an important topic the study of processes induced by Flavour Changing Neutral Current (FCNC) $b \rightarrow s$ transitions, since they provide us with tests of the Standard Model (SM) and constraints for New Physics (NP) scenarios. The observation of $B_s^0 - \bar{B}_s^0$ oscillations at the Tevatron [2] represents a great success in this Physics programme.

Considering the experimental situation, data are available at present for several $b \rightarrow s$ FCNC B meson decays. Together with the inclusive radiative $B \rightarrow X_s\gamma$ branching ratio, the rates of a few exclusive radiative modes, both for charged B : $B^\pm \rightarrow K^{*\pm}\gamma$, $K_1(1270)^\pm\gamma$ and $K_2^*(1430)^\pm\gamma$, both for neutral B : $B^0 \rightarrow K^{*0}\gamma$ and $K_2^*(1430)^0\gamma$ have been measured [3]. Moreover, the branching fractions of $B^\pm \rightarrow K^{(*)\pm}\ell^+\ell^-$ and $B^0 \rightarrow K^{(*)0}\ell^+\ell^-$, with $\ell = e, \mu$ have been determined by the Belle and BaBar collaborations, which have also provided us with preliminary measurements of the lepton forward-backward asymmetry [4, 5] together with the K^* longitudinal helicity fraction in $B \rightarrow K^{*}\ell^+\ell^-$ [6]. Furthermore, upper bounds for $\mathcal{B}(B \rightarrow K^{(*)}\nu\bar{\nu})$ have been established [3]. Even not considering non leptonic $b \rightarrow s$ penguin induced B decays, the interpretation of which is not straightforward, this wealth of measurements has already severely constrained the parameter space of various non standard scenarios. Other tight information could be obtained by more precise data in the modes already

observed, as well as by the measurement of the branching fractions and of the spectra of $B \rightarrow K^{(*)}\nu\bar{\nu}$, and of $B \rightarrow K^{(*)}\tau^+\tau^-$ in which the τ polarization asymmetries are sensitive to Physics beyond SM.

Other processes induced by $b \rightarrow s$ transition involve the hadrons B_s , B_c and Λ_b , the decay modes of which are harder to be experimentally studied, e.g. due to their smaller production rate in b quark hadronization with respect to B mesons. However, at the hadron colliders, in particular at LHC, the number of produced particles is so large that even these processes are expected to be observable, so that their study can contribute to our understanding of Physics of rare transitions within and beyond the Standard Model description of elementary interactions.

In this paper we study a few $b \rightarrow s$ FCNC induced B_s and Λ_b decays, in particular $B_s \rightarrow \phi\gamma, \phi\nu\bar{\nu}$ and $\Lambda_b \rightarrow \Lambda\gamma, \Lambda\nu\bar{\nu}$, in SM and in a New Physics scenario where a single universal extra dimension is considered, the Appelquist, Cheng and Dobrescu (ACD) model [7]. Models with extra dimensions have been proposed as viable candidates to solve some problems affecting SM [8], and within this class of NP models the ACD scenario with a single extra dimension is worth investigating due to its appealing features. Here, we do not discuss in details the various aspects of the model, which have been worked out in [7] and are summarized in [9]-[11]. We only recall that the model consists in a minimal extension of SM in $4 + 1$ dimensions, with the extra dimension compactified to the orbifold S^1/Z_2 . The fifth coordinate y runs from 0 to $2\pi R$, with $y = 0$, $y = \pi R$ fixed points of the orbifold, and R the radius of the orbifold which represents a new physical parameter. All the fields are allowed to propagate in all dimensions, there-

fore the model belongs to the class of *universal* extra dimension scenarios. The four-dimensional description includes SM particles, corresponding to the zero modes of fields propagating in the compactified extra dimension, together with towers of Kaluza-Klein (KK) excitations corresponding to the higher modes. Such fields are imposed to be even under parity transformation in the fifth coordinate $P_5 : y \rightarrow -y$. Fields which are odd under P_5 propagate in the extra dimension without zero modes and correspond to particles with no SM partners.

In addition to 5-d bulk terms, the Lagrangian of the ACD model may also include boundary terms which represent additional parameters of the theory and get renormalized by bulk interactions, although being volume suppressed. A simplifying assumption is that such boundary terms vanish at the cut-off scale, so that a minimal Universal Extra Dimension model can be defined in which the only new parameter with respect to the Standard Model is the radius R of the extra dimension, an important feature as far as the phenomenological investigation of the model is concerned [12].

The masses of KK particles depend on the radius R of the extra dimension. For example, the masses of the KK bosonic modes are given by [7], [9]-[11]:

$$m_n^2 = m_0^2 + \frac{n^2}{R^2} \quad n = 1, 2, \dots \quad (1.1)$$

For small values of R these particles, being more and more massive, decouple from the low energy regime. Thus, within this model candidates for dark matter are available, the Kaluza-Klein (KK) excitations of the photon or of the neutrinos with KK number $n = 1$ [13, 14]. This is related to another property of the ACD model, the conservation of the KK parity $(-1)^j$, with j the KK number [15]. KK parity conservation implies the absence of tree level contributions of Kaluza Klein states to processes taking place at low energy, $\mu \ll 1/R$, a feature which permits to establish a bound: $1/R \geq 250 - 300$ GeV by the analysis of Tevatron run I data [16]. Moreover, the precision measurements of electroweak observables at the LEP collider allow to obtain the bound $1/R \geq 600$ GeV assuming for the Higgs a mass of 115 GeV, a constraint on the extra dimension radius which can be relaxed as low as $1/R \geq 300$ GeV with increasing Higgs mass [17]. Other bounds can be derived invoking cosmological arguments; in Ref.[18] it was found that the region of parameters preferred by cosmological constraints in a single UED scenario corresponds to a Higgs mass between 185 and 245 GeV, to a lightest KK particle between 810 and 1400 GeV and to maximal spitting between the first KK modes of 320 GeV; such constraints come from the diffuse photon spectrum and from the null searches of exotic stable charged particles, assuming that the first excitation of the graviton is the lightest KK particle. These bounds, however, are different if the lightest KK particle is not the first excitation of the graviton.

The fact that KK excitations can influence processes occurring at loop level suggests that FCNC transitions

are particularly suitable for constraining the extra dimension model, providing us with many observables sensitive to the compactification radius R . For this reason, in [9, 10] the effective Hamiltonian governing $b \rightarrow s$ transitions was derived, and inclusive $B \rightarrow X_s \gamma$, $B \rightarrow X_s \ell^+ \ell^-$, $B \rightarrow X_s \nu \bar{\nu}$ transitions, together with the $B_{s(d)}$ mixing, were studied. In particular, it was found that $\mathcal{B}(B \rightarrow X_s \gamma)$ allowed to constrain $1/R \geq 250$ GeV, a bound updated by a more recent analysis based on the NNLO value of the SM Wilson coefficient c_7 (defined in the next section) [19] and on new experimental data to $1/R \geq 600$ GeV at 95% CL, or to $1/R \geq 330$ GeV at 99% CL [20]. Concerning the exclusive modes $B \rightarrow K^* \gamma$, $B \rightarrow K^{(*)} \ell^+ \ell^-$, $B \rightarrow K^{(*)} \nu \bar{\nu}$ and $B \rightarrow K^{(*)} \tau^+ \tau^-$, it was argued that the uncertainty related to the hadronic matrix elements does not obscure the sensitivity to the compactification parameter R , and that current data, in particular the decay rates of $B \rightarrow K^* \gamma$ and $B \rightarrow K^* \ell^+ \ell^-$ ($\ell = e, \mu$) can provide the bound $1/R \geq 300 - 400$ GeV [11, 21].

In case of B_s , the mode $B_s \rightarrow \mu^+ \mu^-$ was recognized as the process receiving the largest enhancement with respect to the SM prediction; however, since the predicted branching fraction is $\mathcal{O}(10^{-9})$, the measurement is very challenging [9]. A modest enhancement was found in $B_s \rightarrow \gamma \gamma$ [22], in $B_s \rightarrow \phi \ell^+ \ell^-$ and in $B_s \rightarrow \ell^+ \ell^- \gamma$ [23]. In the case of Λ_b , for $1/R \simeq 300$ GeV a sizeable effect of the extra dimension was found in $\Lambda_b \rightarrow \Lambda \ell^+ \ell^-$ [24], analogously to what obtained in $B \rightarrow K^{(*)} \ell^+ \ell^-$.

For the modes $B_s \rightarrow \phi \nu \bar{\nu}$, $\Lambda_b \rightarrow \Lambda \gamma$ and $\Lambda_b \rightarrow \Lambda \nu \bar{\nu}$ analyzed in this paper, no data are available at present, while for $B_s \rightarrow \phi \gamma$ a first measurement of the branching fraction has been recently carried out. Our aim is to work out a set of predictions for various observables in the Standard Model, making use also of information obtained at the B factories. Moreover, we consider these processes in the extra dimension scenario studying the dependence on R which could provide us with further ways to constrain such a parameter. To be conservative, we consider the range of $1/R$ starting from $1/R \geq 200$ GeV. In the lowest part of this range the effect of the extra dimension is clearly visible in the various observables we consider; obviously, the bounds previously mentioned must be taken into account in the final considerations.

In the next Section we recall the effective Hamiltonian inducing $b \rightarrow s \gamma$ and $b \rightarrow s \nu \bar{\nu}$ decays in SM and in the ACD model. The case of B_s is considered in Section III, while the Λ_b transitions are the subject of Section IV. The last Section is devoted to the conclusions.

II. $b \rightarrow s \gamma$ AND $b \rightarrow s \nu \bar{\nu}$ EFFECTIVE HAMILTONIANS

In the Standard Model the $b \rightarrow s \gamma$ and $b \rightarrow s \nu \bar{\nu}$ transitions are described by the effective $\Delta B = -1$, $\Delta S = 1$

Hamiltonians

$$H_{b \rightarrow s\gamma} = 4 \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* c_7^{eff}(\mu) O_7(\mu) \quad (2.1)$$

and

$$H_{b \rightarrow s\nu\bar{\nu}} = \frac{G_F}{\sqrt{2}} \frac{\alpha(M_W)}{2\pi \sin^2(\theta_W)} V_{tb} V_{ts}^* \eta_X X(x_t) O_L = c_L O_L \quad (2.2)$$

involving the operators

$$O_7 = \frac{e}{16\pi^2} [m_b(\bar{s}_L \sigma^{\mu\nu} b_R) + m_s(\bar{s}_R \sigma^{\mu\nu} b_L)] F_{\mu\nu} \quad (2.3)$$

and

$$O_L = \bar{s}\gamma^\mu(1 - \gamma_5)b\bar{\nu}\gamma_\mu(1 - \gamma_5)\nu, \quad (2.4)$$

respectively. Eq.(2.1) describes magnetic penguin diagrams, while (2.2) is obtained from Z^0 penguin and box diagrams, with the dominant contribution corresponding to a top quark intermediate state. G_F is the Fermi constant and V_{ij} are elements of the CKM mixing matrix; moreover, $b_{R,L} = \frac{1 \pm \gamma_5}{2} b$, α is the electromagnetic constant, θ_W the Weinberg angle and $F_{\mu\nu}$ denotes the electromagnetic field strength tensor. The function $X(x_t)$ ($x_t = \frac{m_t^2}{M_W^2}$, with m_t the top quark mass) has been computed in [25] and [26]; the QCD factor η_X is close to one, so that we can put it to unity [27, 28].

In eq. (2.1) we use an effective coefficient c_7^{eff} which turns out to be scheme independent and takes into account the mixing between the operators O_8 with O_7 under renormalization group evolution [29].

In the ACD model no operators other than those in (2.1) and (2.2) contribute to $b \rightarrow s\gamma$ and $b \rightarrow s\nu\bar{\nu}$ transitions: the model belongs to the class of Minimal Flavour Violating models, where effects beyond SM are only encoded in the Wilson coefficients of the effective Hamiltonian [9, 10]. KK excitations only modify c_7 and c_L inducing a dependence on the compactification radius R . For large values of $1/R$, due to decoupling of massive KK states, the Wilson coefficients reproduce the Standard Model values, so that the SM phenomenology is recovered. As a general expression, the Wilson coefficients are represented by functions $F(x_t, 1/R)$ generalizing their SM analogues $F_0(x_t)$:

$$F(x_t, 1/R) = F_0(x_t) + \sum_{n=1}^{\infty} F_n(x_t, x_n), \quad (2.5)$$

with $x_n = \frac{m_n^2}{M_W^2}$ and $m_n = \frac{n}{R}$. A remarkable result is that the sum over the KK contributions in (2.5) is finite at the leading order (LO) in all cases as a consequence of a generalized GIM mechanism [9, 10]. When $R \rightarrow 0$ the Standard Model result is obtained since $F(x_t, 1/R) \rightarrow F_0(x_t)$ in that limit.

For $1/R$ of the order of a few hundreds of GeV the coefficients differ from their Standard Model value; in particular c_7 is suppressed, as one can infer considering the expressions collected in [9]-[11] together with the function $X(x_t, 1/R)$. Therefore, the predicted widths and spectra are modified with respect to SM. In case of exclusive decays, it is important to study if this effect is obscured by the hadronic uncertainties, a discussion that we present in the following two Sections for B_s and Λ_b , respectively.

III. $B_s \rightarrow \phi\gamma$ AND $B_s \rightarrow \phi\nu\bar{\nu}$ DECAYS

The description of the decay modes $B_s \rightarrow \phi\gamma$ and $B_s \rightarrow \phi\nu\bar{\nu}$ involves the hadronic matrix elements of the operators appearing in the effective Hamiltonians (2.1)-(2.2). In case of $B_s \rightarrow \phi\gamma$ the matrix element of O_7 can be parameterized in terms of three form factors:

$$\begin{aligned} & \langle \phi(p', \epsilon) | \bar{s}\sigma_{\mu\nu} q^\nu \frac{(1 + \gamma_5)}{2} b | B_s(p) \rangle \\ &= i\epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p^\alpha p'^\beta 2 T_1(q^2) + \\ &+ \left[\epsilon_\mu^* (M_{B_s}^2 - M_\phi^2) - (\epsilon^* \cdot q)(p + p')_\mu \right] T_2(q^2) \\ &+ (\epsilon^* \cdot q) \left[q_\mu - \frac{q^2}{M_{B_s}^2 - M_\phi^2} (p + p')_\mu \right] T_3(q^2), \end{aligned} \quad (3.1)$$

where $q = p - p'$ is the momentum of the photon and ϵ the ϕ meson polarization vector. At zero value of q^2 the condition $T_1(0) = T_2(0)$ holds, so that the $B_s \rightarrow \phi\gamma$ decay amplitude involves a single hadronic parameter, $T_1(0)$. On the other hand, the matrix element of O_L can be parameterized as follows:

$$\begin{aligned} & \langle \phi(p', \epsilon) | \bar{s}\gamma_\mu(1 - \gamma_5)b | B_s(p) \rangle \\ &= \epsilon_{\mu\nu\alpha\beta} \epsilon^{*\nu} p^\alpha p'^\beta \frac{2V(q^2)}{M_{B_s} + M_\phi} - i \left[\epsilon_\mu^* (M_{B_s} + M_\phi) A_1(q^2) \right. \\ &- (\epsilon^* \cdot q)(p + p')_\mu \frac{A_2(q^2)}{M_{B_s} + M_\phi} \\ &- (\epsilon^* \cdot q) \frac{2M_\phi}{q^2} (A_3(q^2) - A_0(q^2)) q_\mu \left. \right] \end{aligned} \quad (3.2)$$

with a relation holding among the form factors A_1 , A_2 and A_3 :

$$A_3(q^2) = \frac{M_{B_s} + M_\phi}{2M_\phi} A_1(q^2) - \frac{M_{B_s} - M_\phi}{2M_\phi} A_2(q^2) \quad (3.3)$$

together with $A_3(0) = A_0(0)$.

The form factors represent a source of uncertainty in predicting the B_s decay rates we are considering. The other parameters are fixed to $m_b = 4.8 \pm 0.2$ GeV and $M_{B_s} = 5.3675 \pm 0.0018$ GeV, together with $\tau_{B_s^0} = (1.466 \pm 0.052) \times 10^{-12}$ s, $V_{tb} = 0.999$ and $V_{ts} = 0.0406 \pm 0.0027$, using the central values and the uncertainties quoted by the Particle Data Group [3].

A. $B_s \rightarrow \phi\gamma$

From the effective Hamiltonian (2.1), together with (2.3) and the matrix element (3.1), it is straightforward to calculate the expression of the $B_s \rightarrow \phi\gamma$ decay rate:

$$\Gamma(B_s \rightarrow \phi\gamma) = \frac{\alpha(0)G_F^2}{8\pi^4} |V_{tb}V_{ts}^*|^2 (m_b^2 + m_s^2) |c_7^{eff}|^2 [T_1(0)]^2 \times M_{B_s}^3 \left(1 - \frac{M_\phi^2}{M_{B_s}^2}\right)^3. \quad (3.4)$$

This expression is useful to relate the branching fraction $\mathcal{B}(B_s \rightarrow \phi\gamma)$ to the measured value of $\mathcal{B}(B_d \rightarrow K^{*0}\gamma)$, since

$$\mathcal{B}(B_s \rightarrow \phi\gamma) = \left(\frac{T_1^{B_s \rightarrow \phi}(0)}{T_1^{B_d \rightarrow K^{*0}}(0)}\right)^2 \left(\frac{M_{B_d}}{M_{B_s}}\right)^3 \times \left(\frac{M_{B_s}^2 - M_\phi^2}{M_{B_d}^2 - M_{K^{*0}}^2}\right)^3 \frac{\tau_{B_s}}{\tau_{B_d}} \mathcal{B}(B_d \rightarrow K^{*0}\gamma) \quad (3.5)$$

where we have indicated which hadronic matrix elements the form factors refer to. Eq.(3.5) shows that, in addition to measured quantities, the crucial quantity to predict $\mathcal{B}(B_s \rightarrow \phi\gamma)$ is the $SU(3)_F$ breaking parameter r defined by

$$\frac{T_1^{B_s \rightarrow \phi}(0)}{T_1^{B_d \rightarrow K^{*0}}(0)} = 1 + r, \quad (3.6)$$

as shown in Fig. 1 for positive values of r , using $\mathcal{B}(B_d \rightarrow K^{*0}\gamma) = (4.1 \pm 0.2) \times 10^{-5}$ and $\tau_{B_d} = (1.530 \pm 0.009) \times 10^{-12}$ s [3], and combining in quadrature the uncertainties of the various quantities in (3.5). Detailed analyses of the range of values within which r can vary are not available, yet. Using $r = 0.048 \pm 0.006$ estimated by Light Cone Sum Rules (LCSR) [30] we obtain the range bounded by the dashed vertical lines in Fig. 1, which allows us to predict:

$$\mathcal{B}(B_s \rightarrow \phi\gamma) = (4.2 \pm 0.3) \times 10^{-5}. \quad (3.7)$$

Notice that the chosen value of r is smaller than an analogous quantity parameterizing the ratio of leptonic constants f_{B_s}/f_{B_d} , estimated as: $r = 0.09 \pm 0.03$ [31].

A compatible value of $\mathcal{B}(B_s \rightarrow \phi\gamma)$ is obtained in SM using the form factor $T_1(0) = (17.45 \pm 1.65) \times 10^{-2}$ determined by LCSR [30], although with a larger uncertainty:

$$\mathcal{B}(B_s \rightarrow \phi\gamma) = (4.1 \pm 1.0) \times 10^{-5}. \quad (3.8)$$

These results must be compared to the measurement:

$$\mathcal{B}(B_s \rightarrow \phi\gamma)_{exp} = (5.7_{-1.5}^{+1.8}(stat)_{-1.1}^{+1.2}(syst)) \times 10^{-5} \quad (3.9)$$

recently carried out by Belle Collaboration in a run at the $\Upsilon(5S)$ peak [32]; this experimental result is affected by a statistical and systematical uncertainty which are expected to be reduced in the near future.

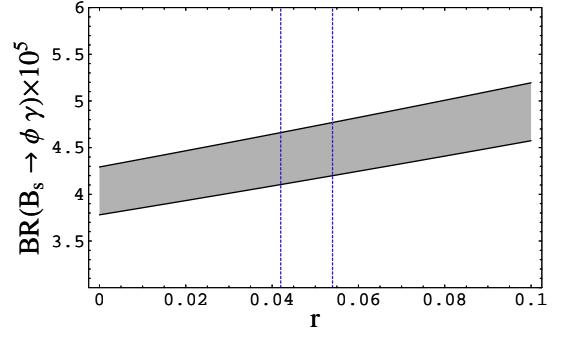


FIG. 1: $\mathcal{B}(B_s \rightarrow \phi\gamma)$ as a function of the $SU(3)_F$ breaking parameter r in the ratio of $B_s \rightarrow \phi$ vs $B_d \rightarrow K^{*0}$ form factors $T_1(0)$ (3.6). The dashed vertical lines show the range of r obtained by LCSR.

In the single extra dimension scenario the modification of the Wilson coefficient c_7^{eff} , corresponding to the variation of the compactification radius R , changes the prediction for $\mathcal{B}(B_s \rightarrow \phi\gamma)$, as shown in Fig. 2 where we plot the branching ratio versus $1/R$. The width of the band reflects the uncertainty on the form factor quoted above, as well as on the parameters m_b and V_{ts} . As for c_7^{eff} , it is affected by the uncertainty due to the higher order corrections in the ACD model. The inclusion of next-to-leading order QCD corrections would require the contribution of two-loop diagrams involving KK gluon corrections which at present is not known. One could estimate the size of this correction by considering the effect of the variation of the matching scale, as done in [20], where it was found that changing this scale between 80 and 320 GeV the value of $\mathcal{B}(B \rightarrow X_s\gamma)$ is affected by an uncertainty not exceeding $|\pm 9\%$ for $1/R$ in the range between 200 and 1500 GeV. For the exclusive modes considered here, such an uncertainty, expected to be similar, must be combined in quadrature with the other errors, in particular with the hadronic uncertainty which is at present the largest one.

We observe that for low values of $1/R$ the branching ratio is reduced with respect to the SM expectation: at, e.g., $1/R = 300$ GeV $\mathcal{B}(B_s \rightarrow \phi\gamma)$ is smaller by 35% as a consequence of the lower value of c_7^{eff} in the ACD model. This effect was already noticed in the analysis of $B \rightarrow X_s\gamma$ and $B \rightarrow K^{*0}\gamma$. For higher values of $1/R$ the lowering of the branching fraction is obscured by the hadronic uncertainty due to the form factor.

B. $B_s \rightarrow \phi\nu\bar{\nu}$

Analogously to the case of $B \rightarrow K^{*0}\nu\bar{\nu}$ [33], for this decay mode it is convenient to separately consider the

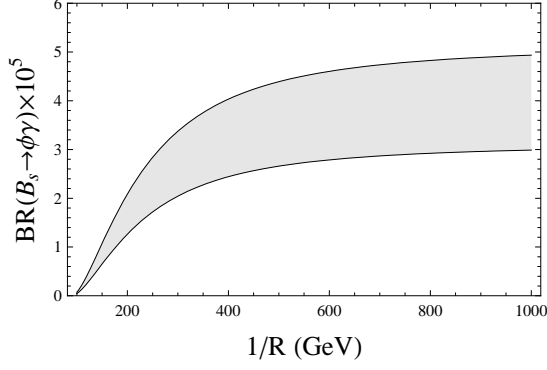


FIG. 2: $\mathcal{B}(B_s \rightarrow \phi\gamma)$ vs the inverse radius $1/R$ of the compactified extra dimension in the ACD model. The Belle measurement is: $\mathcal{B}(B_s \rightarrow \phi\gamma)_{exp} = (5.7^{+1.8}_{-1.5}(stat)^{+1.2}_{-1.1}(syst)) \times 10^{-5}$.

missing energy distributions for longitudinally (L) and transversely (\pm) polarized ϕ mesons:

$$\frac{d\Gamma_L}{dx} = 3 \frac{|c_L|^2 |\vec{p}'|}{24\pi^3 M_\phi^2} [(M_{B_s} + M_\phi)(M_{B_s} E' - M_\phi^2) A_1(q^2) - \frac{2M_{B_s}^2}{M_{B_s} + M_\phi} |\vec{p}'|^2 A_2(q^2)]^2, \quad (3.10)$$

and

$$\frac{d\Gamma_\pm}{dx} = 3 \frac{|\vec{p}'|^2 |c_L|^2}{24\pi^3} \left| \frac{2M_{B_s} |\vec{p}'|}{M_{B_s} + M_\phi} V(q^2) \mp (M_{B_s} + M_\phi) A_1(q^2) \right|^2. \quad (3.11)$$

In (3.10)-(3.11) $x = E_{miss}/M_{B_s}$, with E_{miss} the energy of the neutrino pair (missing energy); q is the momentum transferred to the neutrino pair, \vec{p}' and E' the ϕ three-momentum and energy in the B_s meson rest frame, and the sum over the three neutrino species has been carried out.

The missing energy distributions for polarized and unpolarized ϕ are depicted in Fig. 3 for $1/R = 500$ GeV and in the Standard Model. They are obtained using $B_s \rightarrow \phi$ form factors determined by LCSR [30]:

$$\begin{aligned} A_1(q^2) &= \frac{A_1(0)}{1 - \frac{q^2}{m_{A_1}^2}} \\ A_2(q^2) &= \frac{r_1^{A_2}}{1 - \frac{q^2}{m_{A_2}^2}} + \frac{r_2^{A_2}}{\left(1 - \frac{q^2}{m_{A_2}^2}\right)^2} \\ V(q^2) &= \frac{r_1^V}{1 - \frac{q^2}{m_R^2}} + \frac{r_2^V}{1 - \frac{q^2}{m_V^2}} \end{aligned} \quad (3.12)$$

where $A_1(0) = 0.311 \pm 0.030$ and $m_{A_1}^2 = 36.54$ GeV²; $r_1^{A_2} = -0.054$, $r_2^{A_2} = 0.288$, $A_2(0) = r_1^{A_2} + r_2^{A_2} = 0.234 \pm$

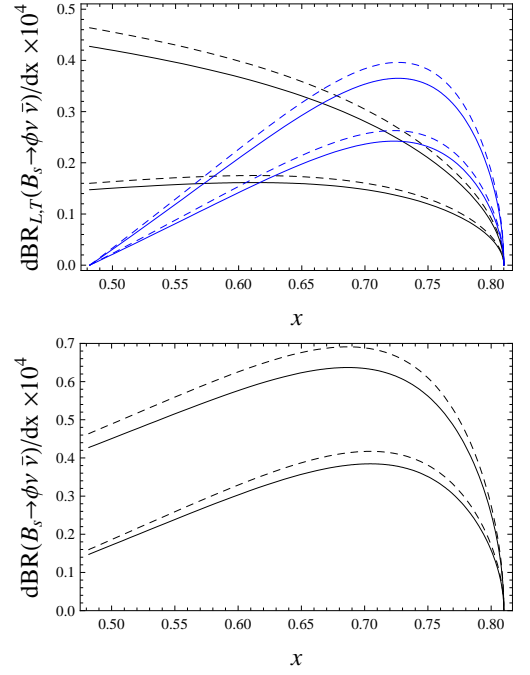


FIG. 3: Missing energy distribution in $B_s \rightarrow \phi\nu\bar{\nu}$ (upper panel) for longitudinally (left curves) and transversally polarized ϕ meson (right curves). The sum over the three neutrino species is understood. The continuous lines bound the region corresponding to SM, the dashed lines the region corresponding to $1/R = 500$ GeV. In the lower panel the missing energy distribution for unpolarized ϕ is depicted (same notations).

0.028 , $m_{A_2}^2 = 48.94$ GeV²; $r_1^V = 1.484$, $r_2^V = -1.049$, $V(0) = r_1^V + r_2^V = 0.434 \pm 0.035$, $m_R = 5.32$ GeV, $m_V^2 = 39.52$ GeV². The effect of the extra dimension consists in a systematic increase of the various distributions in the full range of missing energy; however, the hadronic uncertainty needs to be substantially reduced in order to clearly disentangle deviations from the Standard Model predictions which correspond to this value of $1/R$.

In the SM the branching ratio is predicted:

$$\mathcal{B}(B_s \rightarrow \phi\nu\bar{\nu}) = (1.4 \pm 0.4) \times 10^{-5} \quad (3.13)$$

therefore this mode is within the reach of future experiments, at least as far as the number of produced events is concerned, although the observation of a final state involving a neutrino-antineutrino pair is a challenging task, as observed also in [34]. The dependence of $\mathcal{B}(B_s \rightarrow \phi\nu\bar{\nu})$ on $1/R$ is depicted in Fig. 4, where it is shown that the branching fraction increases for low values of $1/R$: for example, at $1/R = 300$ GeV there is a 23% enhancement with respect to the SM expectation. For larger values of $1/R$ the Standard Model prediction is recovered, the dependence on $1/R$ being obscured by form factor uncertainties.

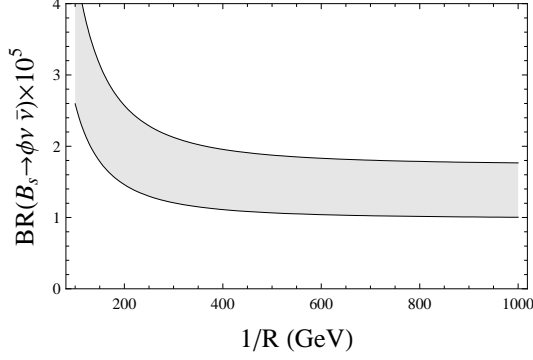


FIG. 4: $\mathcal{B}(B_s \rightarrow \phi \nu \bar{\nu})$ vs the compactification parameter $1/R$ in the ACD model.

IV. $\Lambda_b \rightarrow \Lambda \gamma$ AND $\Lambda_b \rightarrow \Lambda \nu \bar{\nu}$ DECAYS

In case of $\Lambda_b \rightarrow \Lambda$ transitions the hadronic matrix elements of the operators O_7 and O_L in eqs. (2.1) and (2.2) involve a larger number of form factors. As a matter of fact, the various matrix elements can be written as follows:

$$\begin{aligned} & \langle \Lambda(p', s') | \bar{s} i \sigma_{\mu\nu} q^\nu b | \Lambda_b(p, s) \rangle \\ &= \bar{u}_\Lambda [f_1^T(q^2) \gamma_\mu + i f_2^T(q^2) \sigma_{\mu\nu} q^\nu + f_3^T(q^2) q_\mu] u_{\Lambda_b} \end{aligned} \quad (4.1)$$

$$\begin{aligned} & \langle \Lambda(p', s') | \bar{s} i \sigma_{\mu\nu} q^\nu \gamma_5 b | \Lambda_b(p, s) \rangle \\ &= \bar{u}_\Lambda [g_1^T(q^2) \gamma_\mu \gamma_5 + i g_2^T(q^2) \sigma_{\mu\nu} q^\nu \gamma_5 + g_3^T(q^2) q_\mu \gamma_5] u_{\Lambda_b} \end{aligned} \quad (4.2)$$

$$\begin{aligned} & \langle \Lambda(p', s') | \bar{s} \gamma_\mu b | \Lambda_b(p, s) \rangle \\ &= \bar{u}_\Lambda [f_1(q^2) \gamma_\mu + i f_2(q^2) \sigma_{\mu\nu} q^\nu + f_3(q^2) q_\mu] u_{\Lambda_b} \end{aligned} \quad (4.3)$$

$$\begin{aligned} & \langle \Lambda(p', s') | \bar{s} \gamma_\mu \gamma_5 b | \Lambda_b(p, s) \rangle \\ &= \bar{u}_\Lambda [g_1(q^2) \gamma_\mu \gamma_5 + i g_2(q^2) \sigma_{\mu\nu} q^\nu \gamma_5 + g_3(q^2) q_\mu \gamma_5] u_{\Lambda_b}, \end{aligned} \quad (4.4)$$

with u_Λ and u_{Λ_b} the Λ and Λ_b spinors.

At present, a determination of all the form factors in (4.1)-(4.4) is not available. However, it is possible to invoke heavy quark symmetries for the hadronic matrix elements involving an initial spin= $\frac{1}{2}$ heavy baryon comprising a single heavy quark Q and a final $\frac{1}{2}$ light baryon; the heavy quark symmetries reduce to two the number of independent form factors. As a matter of fact, in the

infinite heavy quark limit $m_Q \rightarrow \infty$ and for a generic Dirac matrix Γ one can write [35]:

$$\begin{aligned} & \langle \Lambda(p', s') | \bar{s} \Gamma b | \Lambda_b(p, s) \rangle \\ &= \bar{u}_\Lambda(p', s') \{ F_1(p' \cdot v) + \not{v} F_2(p' \cdot v) \} \Gamma u_{\Lambda_b}(v, s) \end{aligned} \quad (4.5)$$

where $v = \frac{p}{M_{\Lambda_b}}$ is the Λ_b four-velocity. The form factors F_1 and F_2 depend on $p' \cdot v = \frac{M_{\Lambda_b}^2 + M_\Lambda^2 - q^2}{2M_{\Lambda_b}}$ (for convenience we instead consider them as functions of q^2 through this relation). The expression (4.5) for a generic matrix element not only shows that the number of independent form factors is reduced to two, but also that such form factors are universal, since the same functions F_1 and F_2 describe both Λ_b , both Λ_c decays into Λ , envisaging the possibility of relating these two kind of processes if finite quark mass effects, in particular in the charm case, are small. The relations between the form factors in (4.1)-(4.4) and the universal functions in (4.5):

$$\begin{aligned} f_1 &= g_1 = f_2^T = g_2^T = F_1 + \frac{M_\Lambda}{M_{\Lambda_b}} F_2 \\ f_2 &= g_2 = f_3 = g_3 = \frac{F_2}{M_{\Lambda_b}} \\ f_1^T &= g_1^T = q^2 \frac{F_2}{M_{\Lambda_b}} \\ f_3^T &= -(M_{\Lambda_b} - M_\Lambda) \frac{F_2}{M_{\Lambda_b}} \\ g_3^T &= (M_{\Lambda_b} + M_\Lambda) \frac{F_2}{M_{\Lambda_b}} \end{aligned} \quad (4.6)$$

are strictly valid at momentum transfer close to the maximum value $q^2 \simeq q_{max}^2 = (M_{\Lambda_b} - M_\Lambda)^2$. However, we extend their validity to the whole phase space, an assumption which introduces a model dependence in the predictions.

A determination of F_1 and F_2 has been obtained by three-point QCD sum rules in the $m_Q \rightarrow \infty$ limit [36]. In the following we use the expressions for the functions F_1 , F_2 obtained by updating some of the parameters used in [36]. In particular, we use the PDG value of the Λ_b mass: $M_{\Lambda_b} = 5.624 \pm 0.009$ GeV [3]. Moreover, we fix the mass difference $\Delta_{\Lambda_b} = M_{\Lambda_b} - m_b$, together with a constant f_{Λ_b} parameterizing a vacuum-current Λ_b matrix element, to the values computed in the infinite heavy quark mass limit in [37]: $\Delta_{\Lambda_b} = 0.9 \pm 0.1$ GeV and $f_{\Lambda_b} = (2.9 \pm 0.5) \times 10^{-2}$ GeV³. Using these inputs, the obtained form factors $F_{1,2}$ can be parameterized by the expressions:

$$F_{1,2}(q^2) = \frac{F_{1,2}(0)}{1 + a_{1,2} q^2 + b_{1,2} q^4} \quad (4.7)$$

with $F_1(0) = 0.322 \pm 0.015$, $a_1 = -0.0187$ GeV⁻², $b_1 = -1.6 \times 10^{-4}$ GeV⁻⁴, and $F_2(0) = -0.054 \pm 0.020$, $a_2 = -0.069$ GeV⁻², $b_2 = 1.5 \times 10^{-3}$ GeV⁻⁴.

A few remarks are in order. First, the q^2 (or $p' \cdot v$) dependence of the two form factors turns out to be different, at odds with what has been assumed in various analyses where the same q^2 dependence for F_1 and F_2 is argued [38]. Second, while F_1 and F_2 are monotonic in q^2 , their dependence is different from the simple or multiple pole behaviour assumed in other analyses, with pole mass fixed by vector meson dominance (VMD) arguments. Finally, F_2 is different from zero. This is noticeable, even though in various decay rates the terms involving F_2 appear together with the suppressing factor $1/M_{\Lambda_b}$. An analysis of the helicity amplitudes in $\Lambda_c \rightarrow \Lambda \ell \nu$ carried out by the CLEO collaboration measuring various angular distributions in this decay process demonstrated that F_2 is not vanishing [39]. The result, based on the assumption that the heavy quark limit can be applied in the charm case and on the hypothesis the both F_1 and F_2 have the same dipolar q^2 dependence, is: $\frac{F_2(q^2)}{F_1(q^2)} = -0.35 \pm 0.04 \pm 0.04$ if the pole mass is fixed to $M_{pole} = M_{D_s^*}$, or $\frac{F_2(q^2)}{F_1(q^2)} = -0.31 \pm 0.05 \pm 0.04$ in correspondence to a fitted pole mass $M_{pole} = 2.21 \pm 0.08 \pm 0.14$ GeV.

The idea of using measurements of observables in semileptonic $\Lambda_c \rightarrow \Lambda \ell \nu$ transitions to determine the universal form factors F_1 and F_2 , with the aim of employing them to describe several Λ_b transitions, was proposed in [40]. However, in such a proposal the q^2 dependence of the two form factors must be assumed. If $F_{1,2}$ have both a dipolar q^2 dependence, using the ratio $\frac{F_2}{F_1}$ determined by the CLEO collaboration and the experimental value of $\mathcal{B}(\Lambda_c \rightarrow \Lambda \ell \nu)$, one gets: $F_1(q_{max}^2) = 1.1 \pm 0.2$. Following the strategy proposed in [40] it is necessary to extrapolate F_1 to the full kinematical range allowed in $\Lambda_b \rightarrow \Lambda$ transitions by the assumed q^2 dependence, and apply the result to $\Lambda_b \rightarrow \Lambda \gamma$ and $\Lambda_b \rightarrow \Lambda \nu \bar{\nu}$. We refrain from applying such a procedure, since the momentum range where the extrapolation is required is very wide and, more importantly, the assumption that finite charm mass effects are negligible in case of Λ_c should be justified.

It is worth mentioning that in the $m_b \rightarrow \infty$ limit and in the large recoil regime for the light hadron ($q^2 \simeq 0$ or $E = v \cdot p' \rightarrow \infty$), the relation

$$\frac{F_2(0)}{F_1(0)} = -\frac{M_\Lambda}{2E} \quad (4.8)$$

can be derived using the Large Energy Effective Theory/SCET framework at the leading order in the $\frac{1}{m_b}$ and $\frac{1}{E}$ expansion [41]. Direct calculations of $F_{1,2}$ in this limit have not been done, yet; the model (4.7) gives a ratio of form factors compatible with (4.8).

Admittedly, the knowledge of Λ_b form factors deserves a substantial improvement; in the meanwhile, we use in our analysis the form factors in (4.7) stressing that the uncertainties we attach to the various predictions only

take into account the errors of the parameters of the expressions used for F_1 and F_2 .

A. $\Lambda_b \rightarrow \Lambda \gamma$

The expression of the $\Lambda_b \rightarrow \Lambda \gamma$ decay rate:

$$\Gamma(\Lambda_b \rightarrow \Lambda \gamma) = \frac{\alpha(0)G_F^2|V_{tb}V_{ts}^*|^2m_b^2}{32\pi^4}|c_7^{eff}|^2M_{\Lambda_b}^3 \times \left(1 - \frac{M_\Lambda^2}{M_{\Lambda_b}^2}\right)^3 \left(F_1(0) + \frac{M_\Lambda}{M_{\Lambda_b}}F_2(0)\right)^2 \quad (4.9)$$

is useful to relate also this mode to the observed $B_d \rightarrow K^{*0} \gamma$ transition:

$$\mathcal{B}(\Lambda_b \rightarrow \Lambda \gamma) = \left(\frac{F_1(0)}{T_1^{B_d \rightarrow K^{*0}}(0)}\right)^2 \left(1 + \frac{M_\Lambda}{M_{\Lambda_b}} \frac{F_2(0)}{F_1(0)}\right)^2 \times \left(\frac{M_{B_d}}{M_{\Lambda_b}}\right)^3 \left(\frac{M_{\Lambda_b}^2 - M_\Lambda^2}{M_{B_d}^2 - M_{K^{*0}}^2}\right)^3 \frac{\tau_{\Lambda_b}}{4\tau_{B_d}} \mathcal{B}(B_d \rightarrow K^{*0} \gamma) . \quad (4.10)$$

Using $\tau_{\Lambda_b} = (1.23 \pm 0.074) \times 10^{-12} s$ [3], together with the ratio of form factors $\frac{F_2(0)}{F_1(0)} = -0.17 \pm 0.06$ and $\frac{F_1(0)}{T_1^{B_d \rightarrow K^{*0}}(0)} = 1.9 \pm 0.2$ obtained from (4.7) and the form factor T_1 in [30], we predict: $\mathcal{B}(\Lambda_b \rightarrow \Lambda \gamma) = (3.4 \pm 0.7) \times 10^{-5}$. The same result with a larger uncertainty comes from using T_1 determined in [42]. Using (4.9) and the form factors (4.7), we get:

$$\mathcal{B}(\Lambda_b \rightarrow \Lambda \gamma) = (3.1 \pm 0.6) \times 10^{-5} . \quad (4.11)$$

The result suggests that this process is within the reach of LHC experiments. As for the effect of the extra dimension in modifying the decay rate, in Fig. 5 we show how $\mathcal{B}(\Lambda_b \rightarrow \Lambda \gamma)$ depends on $1/R$. Analogously to $B \rightarrow K^* \gamma$ and $B_s \rightarrow \phi \gamma$ transitions, the branching fraction is suppressed for low values of $1/R$, being 30% smaller for $1/R = 300$ GeV.

B. $\Lambda_b \rightarrow \Lambda \nu \bar{\nu}$

Even for this mode it is useful to consider the missing energy distribution in the variable $x = E_{miss}/M_{\Lambda_b}$ with E_{miss} the energy of the neutrino - antineutrino pair:

$$\frac{d\Gamma}{dx} = \frac{|c_L|^2|\vec{p}'|}{4\pi^3} \left\{ \left[(M_{\Lambda_b}^2 - M_\Lambda^2)^2 + q^2(M_{\Lambda_b}^2 + M_\Lambda^2 - 2q^2) \right] \times \left[F_1 + \frac{M_\Lambda}{M_{\Lambda_b}}F_2 \right]^2 \right\}$$

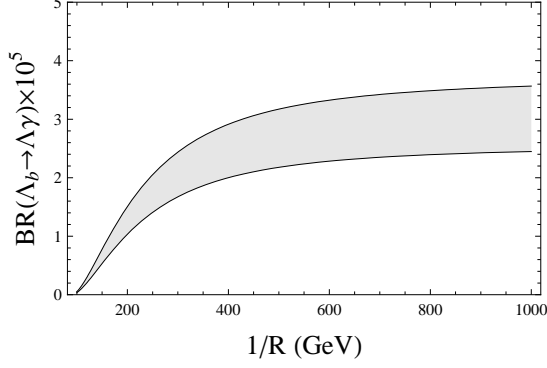


FIG. 5: $\mathcal{B}(\Lambda_b \rightarrow \Lambda\gamma)$ vs $1/R$. The uncertainty shown by the dark band is mainly due to the errors on the form factors of the model (4.7).

$$+ \left[2q^2(M_{\Lambda_b}^2 - M_{\Lambda}^2)^2 - q^4(M_{\Lambda_b}^2 + M_{\Lambda}^2 + q^2) \right] \left(\frac{F_2}{M_{\Lambda_b}} \right)^2 \\ + 6M_{\Lambda}q^2(M_{\Lambda_b}^2 - M_{\Lambda}^2 + q^2) \left[F_1 + \frac{M_{\Lambda}}{M_{\Lambda_b}} F_2 \right] \frac{F_2}{M_{\Lambda_b}} \Big\}. \quad (4.12)$$

In Fig. 6 we plot such a distribution in the SM and for $1/R = 500$ GeV, showing the differences corresponding to such a value of the compactification radius. The branching ratio, which in SM is expected to be [43]:

$$\mathcal{B}(\Lambda_b \rightarrow \Lambda\nu\bar{\nu}) = (6.7 \pm 1.3) \times 10^{-6} \quad (4.13)$$

depends on $1/R$ as shown in Fig. 7. When $1/R$ decreases the branching ratio increases of about 20% for $1/R \simeq 300$ GeV.

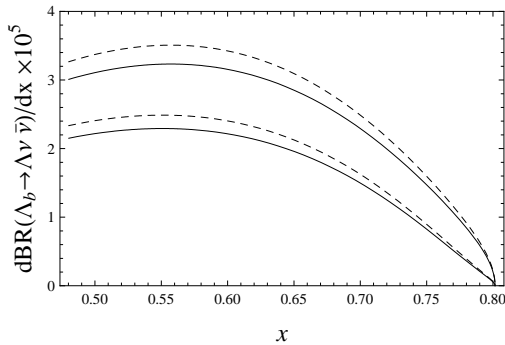


FIG. 6: Missing energy distribution for $\Lambda_b \rightarrow \Lambda\nu\bar{\nu}$ in the Standard Model (continuous lines) and for $1/R = 500$ GeV (dashed lines).

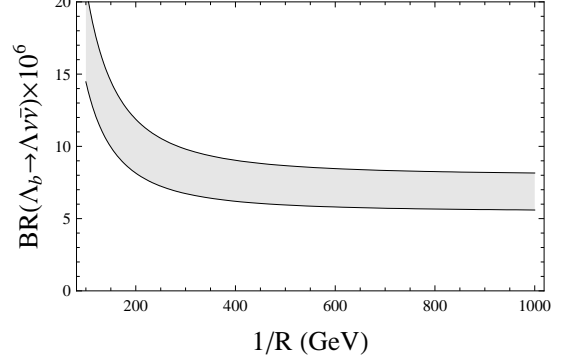


FIG. 7: $\mathcal{B}(\Lambda_b \rightarrow \Lambda\nu\bar{\nu})$ versus $1/R$.

V. CONCLUSIONS

We have studied how a single universal extra dimension could have an impact on several loop induced B_s and Λ_b decays. The analysis of processes involving these two particles will be among the main topics in the investigations at the hadron colliders, especially at LHC. Since a few of the modes we have considered are difficult to reconstruct in a "hostile" environment represented by hadron collisions, we believe that the predictions we have worked out are useful to elaborate the measurement strategies. From the theoretical point of view, we have found that hadronic uncertainties due to the form factors in exclusive decays are not large in case of B_s , where $SU(3)_F$ symmetry is also useful to exploit other measurements carried out at the B factories. As for Λ_b , the situation is more uncertain. Calculations made in the infinite heavy quark limit should be corroborated by the analysis of finite heavy quark mass effects, and the various dependences on the momentum transfer should be confirmed. Such analyses deserve a dedicated effort.

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